

Intro to Trigonometry:

Key

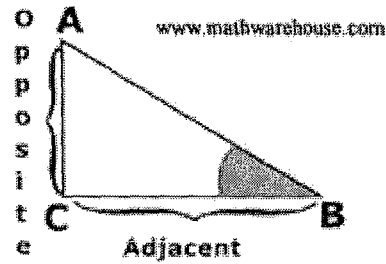
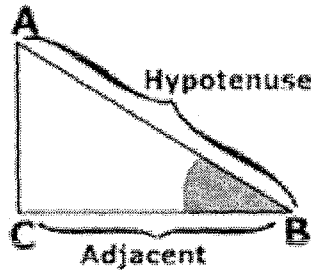
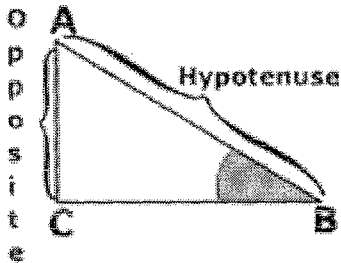
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First, a review of SOH-CAH-TOA:

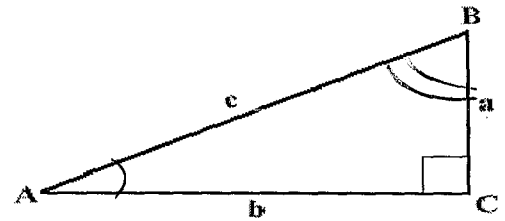
$$\sin(B) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(B) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(B) = \frac{\text{opposite}}{\text{adjacent}}$$



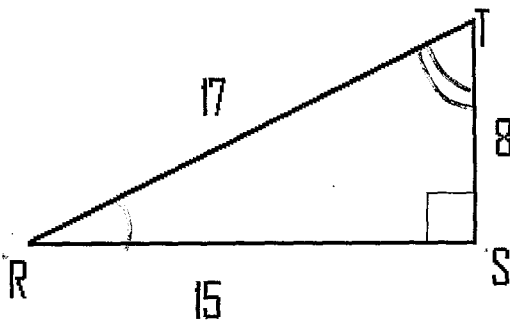
Write the ratios for each of the following functions below:



$$\begin{aligned} \text{sine } A &= \frac{a}{c} \\ \text{cosine } A &= \frac{b}{c} \\ \text{tangent } A &= \frac{a}{b} \end{aligned}$$

$$\begin{aligned} \text{Sine } B &= \frac{b}{c} \\ \text{cosine } B &= \frac{a}{c} \\ \text{tangent } B &= \frac{b}{a} \end{aligned}$$

Give the value of each ratio as a fraction.

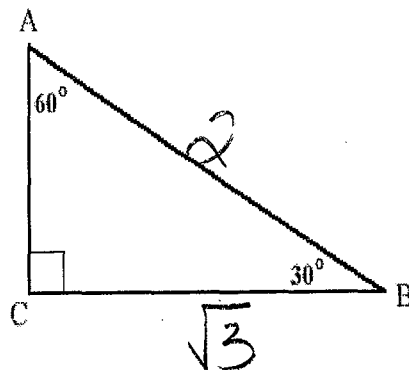
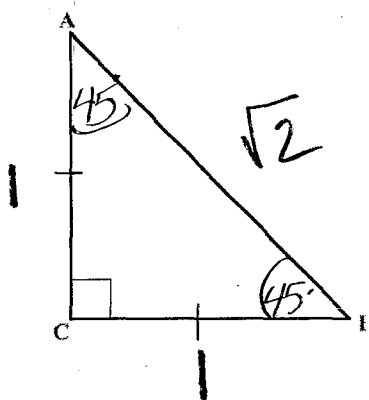


$$\text{Tan } T = \frac{15}{8}$$

$$\begin{aligned} \text{Sin } R &= \frac{8}{17} \\ \text{Cos } R &= \frac{15}{17} \\ \text{Tan } R &= \frac{8}{15} \\ \text{Sin } T &= \frac{15}{17} \\ \text{Cos } T &= \frac{8}{17} \end{aligned}$$

$$x : x : x\sqrt{2} \quad x : x\sqrt{3} : 2x$$

There are two special triangles you need to know, 45-45-90 and 30-60-90 triangles. Do you remember the ratios???



Fill in the table of common trig values based off of the special right triangles above:

	30°	45°	60°
Sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cosine	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
Tangent	$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Evaluate the following:

1) $\sin 60 + \cos 30 =$

$$\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

2) $(\tan 60)(\tan 30) =$

$$\frac{\sqrt{3}}{1} \cdot \frac{\sqrt{3}}{3} = \frac{3}{3} = 1$$

3) $\tan 45 + 2\cos 60 =$

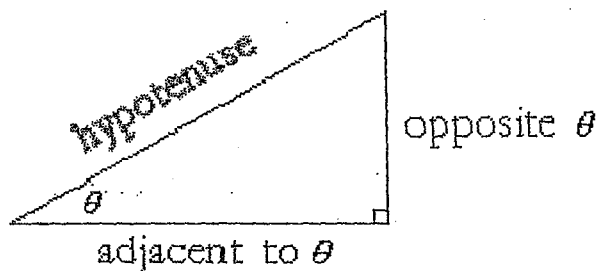
$$1 + 2\left(\frac{1}{2}\right) = 2$$

4) $(\sin 45)^2 + (\cos 45)^2 =$

$$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$

Key

The SIX Trig Functions – That's right I said SIX!



sine of $\theta = \frac{O}{H}$	cosecant of θ : $\csc \theta = \frac{H}{O}$
cosine of $\theta = \frac{A}{H}$	secant of θ : $\sec \theta = \frac{H}{A}$
tangent of $\theta = \frac{O}{A}$	cotangent of θ : $\cot \theta = \frac{A}{O}$

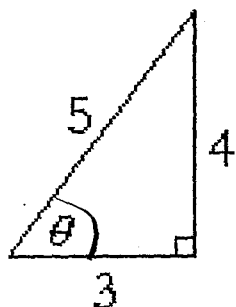
Notice that each ratio (function) in the right-hand column is the reciprocal ratio, of the ratio in the left-hand column.

The reciprocal of $\sin \theta$ is $\csc \theta$; and vice-versa

The reciprocal of $\cos \theta$ is $\sec \theta$

And the reciprocal of $\tan \theta$ is $\cot \theta$

1) The sides of a right triangle are in the ratio 3:4:5, as shown. Name and evaluate the **six** trigonometric functions of angle θ .



$$\sin \theta = \frac{4}{5}$$

$$\cos \theta = \frac{3}{5}$$

$$\tan \theta = \frac{4}{3}$$

$$\csc \theta = \frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

2) Given the coordinates of point P(-6,8)

find the ratio of all six trig functions:

$$\sin \theta = \frac{8}{10}$$

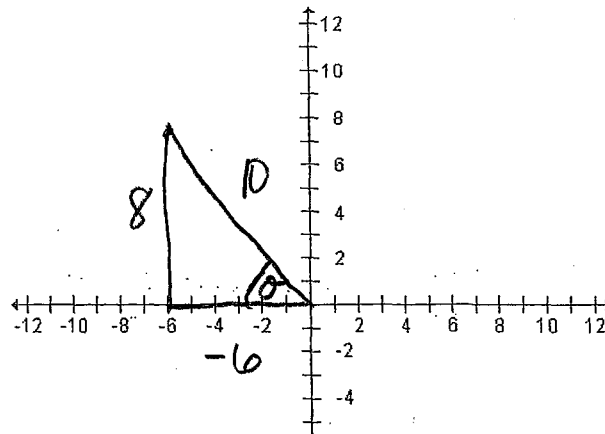
$$\cos \theta = \frac{-6}{10}$$

$$\tan \theta = \frac{8}{-6}$$

$$\csc \theta = \frac{10}{8}$$

$$\sec \theta = \frac{10}{-6}$$

$$\cot \theta = \frac{-6}{8}$$



3) Name the Quadrant in which $\angle B$ must lie.



a) $\cot B < 0$ and $\sin B > 0$
 $\tan \ominus$ $\sin \oplus$ II

b) $\csc B < 0$ and $\sec B > 0$
 $\sin \ominus$ $\cos \oplus$

4) Evaluate the following

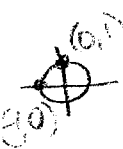


1) $\sec 240^\circ = -\cos 60^\circ = -\frac{1}{2} \rightarrow \boxed{-2}$



2) $\csc 120^\circ = \frac{1}{\sin 60^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \rightarrow \boxed{\frac{2\sqrt{3}}{3}}$

3) $(\tan 45^\circ)(\sec 30^\circ) = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \rightarrow \boxed{\frac{2\sqrt{3}}{3}}$



4) $\csc 90^\circ + \sec 180^\circ = 1 + (-1) = 0$
 "y" "x"
 recip: $1 + -1 = \boxed{0}$

5) $\cot^2 60^\circ = (\frac{1}{\sqrt{3}})^2 = \frac{1}{3} \rightarrow \boxed{\frac{1}{3}}$

Angles formed from the Unit Circle

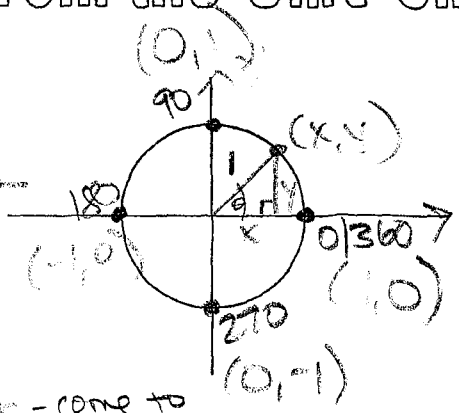
Given a point (x,y) on the unit circle:

$X = \cos \theta$

$Y = \sin \theta$

$\frac{Y}{X} = \tan \theta$

$x^2 + y^2 = 1^2$
 $\cos^2 \theta + \sin^2 \theta = 1 \rightarrow$ on back - come to later



$\cos \theta = \frac{x}{1}$

$\sin \theta = \frac{y}{1}$

$\tan \theta = \frac{y}{x}$

In which Quadrants are the Sine, Cosine, and Tangent Positive?

	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
Sine	+	+	-	-
Cosine	+	-	-	+
Tangent	+	-	+	-

A S T C

How about the trig values for the Quadrantal angles on the Unit Circle?

$\cos 0 = \underline{1}$	$\sin 0 = \underline{0}$	$\tan 0 = \frac{0}{1} = \underline{0}$
$\cos 90 = \underline{0}$	$\sin 90 = \underline{1}$	$\tan 90 = \frac{1}{0} = \underline{\text{urd}}$
$\cos 180 = \underline{-1}$	$\sin 180 = \underline{0}$	$\tan 180 = \frac{0}{-1} = \underline{0}$
$\cos 270 = \underline{0}$	$\sin 270 = \underline{-1}$	$\tan 270 = \frac{-1}{0} = \underline{\text{urd}}$
$\cos 360 = \underline{1}$	$\sin 360 = \underline{0}$	$\tan 360 = \frac{0}{1} = \underline{0}$

X
 Y
 $\frac{Y}{X}$

\rightarrow x s that terminate on x or y axis

Examples

Name which quadrant θ must lie when...

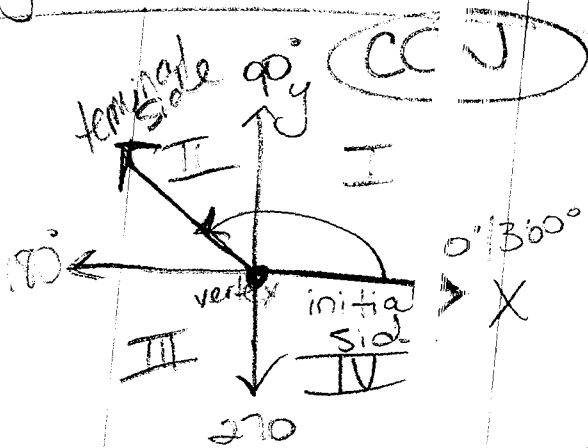
1) $\cos \theta > 0$ and $\sin \theta > 0$ I

2) $\cos \theta < 0$ and $\sin \theta < 0$ III

$\therefore \tan \theta$

Sketching AS

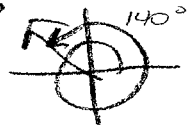
Angles in Standard Position:



ex: 1) 240°



2) 500°



3) -50°



500° and 50°

co-termined AS

b/c they have

same terminal side, found by adding / subtracting multiples of 360°

Pyt. Id. Practice

① Find $\sec \theta$ using Pyt. Id. if $\sin \theta = \frac{3}{5}$, θ is in Q II
 $\left(\frac{3}{5}\right)^2 + \cos^2 \theta = 1$ $\csc \theta = \frac{5}{3}$ Q II

$$\frac{9}{25} + \cos^2 \theta = 1 \quad \frac{25}{25}$$

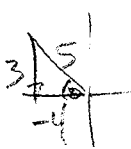
$$\cos^2 \theta = \frac{16}{25}$$

S/A
T/C

$$\sqrt{\cos^2 \theta} = \frac{4}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\boxed{\sec \theta = \frac{5}{4}}$$



② Find $\sin \theta$ if θ is in Q II & $\cos \theta = \frac{2}{7}$

$$\sin^2 \theta + \left(\frac{2}{7}\right)^2 = 1$$

$$\sin^2 \theta + \frac{4}{49} = 1 = \frac{49}{49}$$

$$-\frac{4}{49} \quad -\frac{4}{49}$$

$$\sqrt{\sin^2 \theta} = \frac{\sqrt{45}}{7}$$

$$\sin \theta = \pm \frac{3\sqrt{5}}{7}$$

S/A
T/C

$$\boxed{\sin \theta = \frac{-3\sqrt{5}}{7}}$$

name

Key

Date

Period

Find the exact value of each of the remaining trigonometric functions of θ .

1) $\sin \theta = \frac{12}{13}$, θ in Quadrant II

$\cos = -\frac{5}{13}$ $\sec = \frac{13}{5}$
 $\tan = -\frac{12}{5}$ $\cot = \frac{5}{12}$
 $\csc = \frac{13}{12}$ $\theta = 113^\circ$

2) $\cos \theta = \frac{3}{5}$, θ in Quadrant IV

$\sin = -\frac{4}{5}$
 $\tan = -\frac{4}{3}$
 $\sec = \frac{5}{3}$
 $\csc = -\frac{5}{4}$
 $\cot = -\frac{3}{4}$
 $\theta = 307^\circ$

3) $\cos \theta = -\frac{4}{5}$, θ in Quadrant III

$\sin = -\frac{3}{5}$
 $\tan = \frac{3}{4}$
 $\sec = -\frac{5}{4}$
 $\csc = -\frac{5}{3}$
 $\cot = \frac{4}{3}$
 $\theta = 217^\circ$

4) $\sin \theta = -\frac{5}{13}$, θ in Quadrant III

$\cos = -\frac{12}{13}$
 $\tan = \frac{5}{12}$
 $\csc = -\frac{13}{5}$
 $\sec = -\frac{13}{12}$
 $\cot = \frac{12}{5}$
 $\theta = 203^\circ$

5) $\sin \theta = \frac{5}{13}$, $90^\circ < \theta < 180^\circ$

$\cos = -\frac{12}{13}$
 $\tan = -\frac{5}{12}$
 $\csc = \frac{13}{5}$
 $\sec = -\frac{13}{12}$
 $\cot = -\frac{12}{5}$
 $\theta = 157^\circ$

6) $\cos \theta = \frac{4}{5}$, $270^\circ < \theta < 360^\circ$

$\sin = -\frac{3}{5}$ $\csc = -\frac{5}{3}$
 $\tan = -\frac{3}{4}$ $\cot = -\frac{4}{3}$
 $\sec = \frac{5}{4}$ $\theta = 323^\circ$

7) $\cos \theta = -\frac{1}{3}$, $180^\circ < \theta < 270^\circ$

$1 + x^2 = 9$
 $x^2 = 8$
 $x = 2\sqrt{2}$
 $\sin = \frac{2\sqrt{2}}{3}$ $\csc = \frac{3}{2\sqrt{2}}$
 $\tan = \frac{2\sqrt{2}}{1}$ $\cot = \frac{1}{2\sqrt{2}}$
 $\theta = 222^\circ$

8) $\sin \theta = -\frac{2}{3}$, $180^\circ < \theta < 270^\circ$

$4 + x^2 = 9$
 $x = \sqrt{5}$
 $\cos = -\frac{\sqrt{5}}{3}$ $\csc = \frac{3}{2}$
 $\tan = \frac{2}{\sqrt{5}}$ $\cot = \frac{\sqrt{5}}{2}$
 $\sec = -\frac{3}{\sqrt{5}}$
 $\theta = 222^\circ$

9) $\sin \theta = \frac{2}{3}$, $\tan \theta < 0$

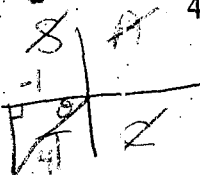
$4 + x^2 = 9$
 $x = \sqrt{5}$
 $\cos = -\frac{\sqrt{5}}{3}$ $\csc = \frac{3}{2}$
 $\tan = -\frac{2}{\sqrt{5}}$ $\cot = -\frac{\sqrt{5}}{2}$
 $\sec = -\frac{3}{\sqrt{5}}$
 $\theta = 138^\circ$

$$1+x^2=16$$

$$x^2=15$$

$$\text{ref } \alpha = 76^\circ \quad \theta = 256^\circ$$

$$\cos \theta = -\frac{1}{4}, \tan \theta > 0$$



$$\sin = \frac{\sqrt{15}}{4}$$

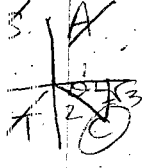
$$\tan = \frac{\sqrt{15}}{-1} = -\sqrt{15}$$

$$\sec = -4$$

$$\csc = \frac{4}{\sqrt{15} \cdot \sqrt{15}} = \frac{4\sqrt{15}}{15}$$

$$\cot = \frac{1}{\sqrt{15} \cdot \sqrt{15}} = \frac{\sqrt{15}}{15}$$

$$11) \sec \theta = 2, \sin \theta < 0, \cos = -\frac{1}{2}$$



$$\sin = -\frac{\sqrt{3}}{2}$$

$$\tan = -\sqrt{3}$$

$$\csc = -\frac{2}{\sqrt{3}}$$

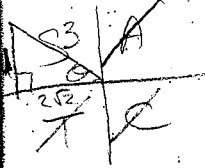
$$\sec = -2$$

$$\cot = -\frac{1}{\sqrt{3}}$$

$$\alpha = 60^\circ$$

$$\theta = 300^\circ$$

$$12) \csc \theta = 3, \cot \theta < 0$$



$$\cos = \frac{2\sqrt{2}}{3}$$

$$\tan = \frac{1 \cdot \sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$$

$$\sec = \frac{3}{2\sqrt{2}}$$

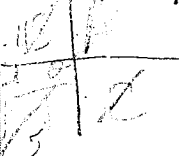
$$\csc = 3$$

$$\cot = \frac{2\sqrt{2}}{\sqrt{2}} = 2$$

$$\text{ref } \alpha = 19^\circ$$

$$\theta = 161^\circ$$

$$13) \tan \theta = \frac{3}{4}, \sin \theta < 0$$



$$\sin = -\frac{3}{5}$$

$$\cos = -\frac{4}{5}$$

$$\tan = \frac{3}{4}$$

$$\csc = -\frac{5}{3}$$

$$\sec = -\frac{5}{4}$$

$$\cot = \frac{4}{3}$$

$$\text{ref } \alpha = 37^\circ$$

$$\theta = 217^\circ$$

$$14) \cot \theta = \frac{4}{3}, \cos \theta < 0$$

$$\tan = \frac{3}{4}$$

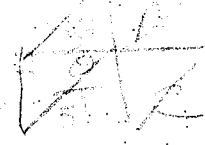
$$\sin = -\frac{3}{5}$$

$$\cos = -\frac{4}{5}$$

$$\csc = -\frac{5}{3}$$

$$\sec = -\frac{5}{4}$$

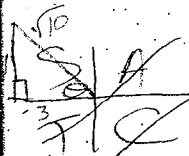
$$\cot = \frac{4}{3}$$



$$\text{ref } \alpha = 37^\circ$$

$$\theta = 217^\circ$$

$$15) \tan \theta = -\frac{1}{3}, \sin \theta > 0$$



$$\sin = \frac{1}{\sqrt{10}}$$

$$\cos = -\frac{3}{\sqrt{10}}$$

$$\tan = -\frac{1}{3}$$

$$\csc = \sqrt{10}$$

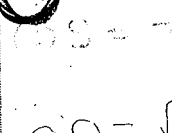
$$\sec = -\sqrt{10}$$

$$\cot = -3$$

$$\text{ref } \alpha = 18^\circ$$

$$\theta = 162^\circ$$

$$16) \sec \theta = -2, \tan \theta > 0$$



$$\sin = -\frac{\sqrt{3}}{2}$$

$$\tan = \sqrt{3}$$

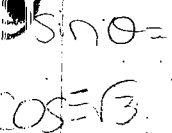
$$\csc = -\frac{2}{\sqrt{3}}$$

$$\cot = \frac{1}{\sqrt{3}}$$

$$\text{ref } \alpha = 60^\circ$$

$$\theta = 240^\circ$$

$$17) \csc \theta = -2, \tan \theta > 0$$



$$\sin = -\frac{1}{2}$$

$$\cos = \frac{\sqrt{3}}{2}$$

$$\tan = -\frac{1}{\sqrt{3}}$$

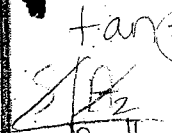
$$\sec = \frac{2}{\sqrt{3}}$$

$$\cot = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\text{ref } \alpha = 30^\circ$$

$$\theta = 210^\circ$$

$$18) \cot \theta = -2, \sec \theta > 0$$



$$\sin = -\frac{1}{\sqrt{5}}$$

$$\cos = \frac{2}{\sqrt{5}}$$

$$\tan = -\frac{1}{2}$$

$$\csc = -\sqrt{5}$$

$$\sec = \frac{\sqrt{5}}{2}$$

$$\cot = -2$$

$$\text{ref } \alpha = 27^\circ$$

$$\theta = 333^\circ$$

$$\sec = \frac{5 \cdot \sqrt{5}}{2 \cdot \sqrt{5}} = \frac{5}{2}$$

Converting from radians to degrees and Vice Vera....

- Converting Degrees \rightarrow Radians: Multiply the angle in degrees by $\frac{\pi \text{ radian}}{180^\circ}$

1) Convert 45 degrees to radians. $45^\circ \cdot \frac{\pi}{180^\circ} = \frac{45\pi}{180} = \frac{\pi}{4}$

2) Convert 90 degrees to radians. $\frac{\pi}{2}$

3) Convert 135 degrees to radians. $135^\circ \cdot \frac{\pi}{180} = \frac{3\pi}{4}$

Converting Radians \rightarrow Degrees: $\frac{180^\circ}{\pi \text{ radians}}$

1. Convert $\frac{5\pi}{9}$ radians to degrees. $\frac{5\pi}{9} \cdot \frac{180^\circ}{\pi} = 100^\circ$

2. Convert 4π radians to degrees. $4\pi \cdot \frac{180^\circ}{\pi} = 720^\circ$

(2 circles)
 $\rightarrow 10 \rightarrow \approx 573^\circ$

Find the Exact value of the following:

* 1) $\tan \frac{8\pi}{10} = \frac{y}{x} = \frac{0}{1} = 0$

2) $\sin \frac{\pi}{3} + \cos \frac{\pi}{6} =$

$\sin 60^\circ + \cos 30^\circ$

$\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

Name Key

Date _____ Radian Measure

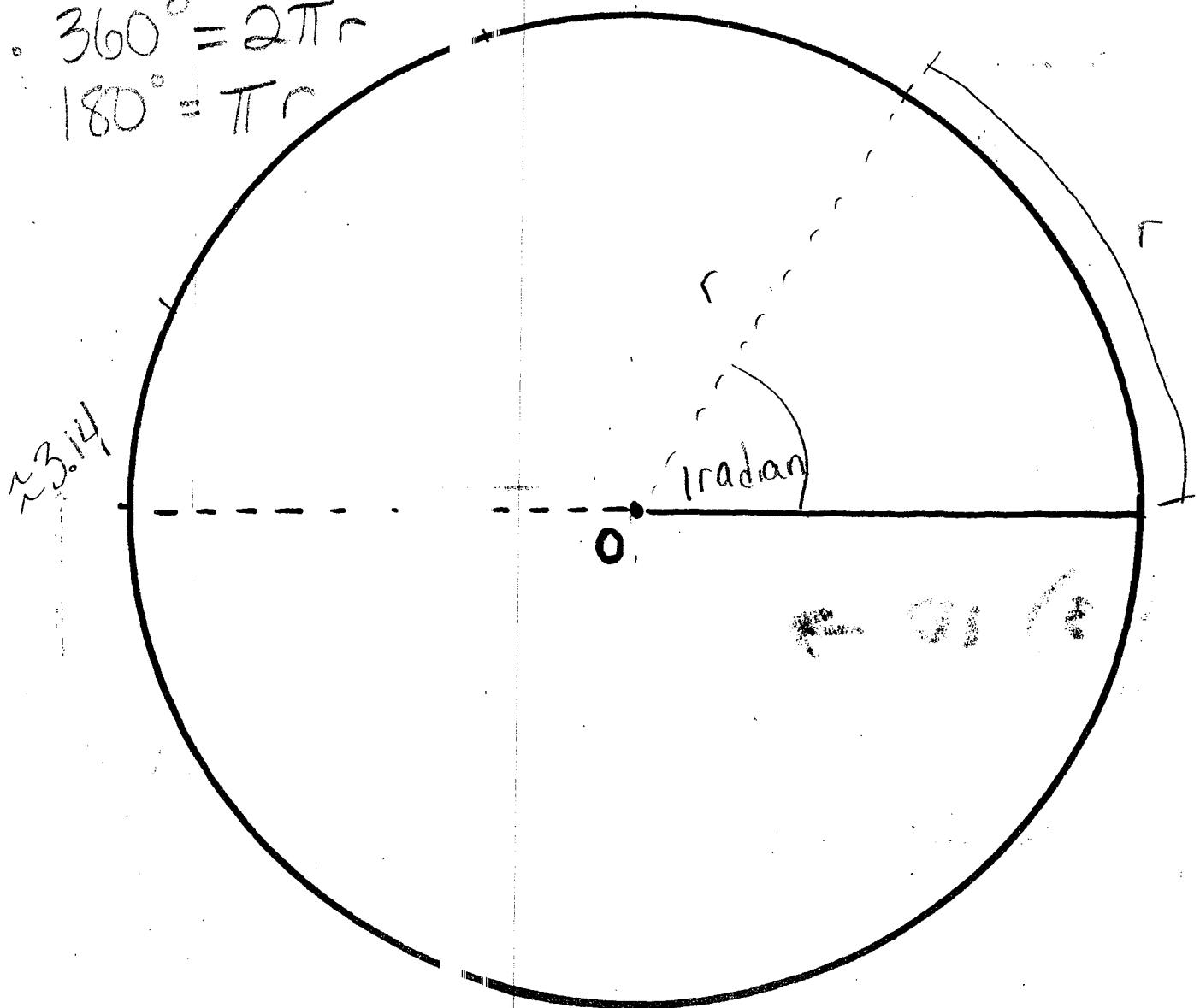
Class Notes:

One radian is the measure of an \angle in standard position whose terminal side intercepts an arc of length r .

B/c $C = 2\pi r$, there are 2π (approx 6.28) radians in a full circle.

$$\therefore 360^\circ = 2\pi r$$

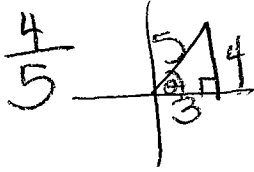
$$180^\circ = \pi r$$



Name: Key

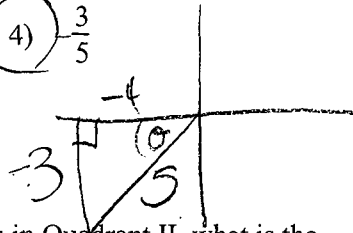
F.TF.C.8: Determining Trigonometric Functions 1a

1 If x is a positive acute angle and $\cos x = \frac{3}{5}$, find the value of $\sin x$.



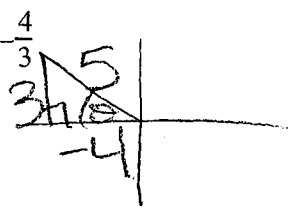
2 If $\cos x = -\frac{4}{5}$ and $\tan x > 0$, the value of $\sin x$ is?

- 1) $\frac{3}{5}$ 2) $\frac{5}{3}$ 3) $-\frac{5}{3}$ 4) $-\frac{3}{5}$



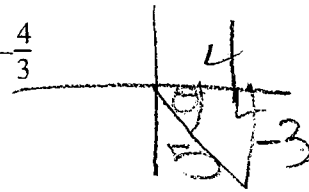
3 If $\cos \theta = -\frac{4}{5}$ and θ lies in Quadrant II, what is the value of $\tan \theta$?

- 1) $\frac{3}{4}$ 2) $\frac{4}{3}$ 3) $-\frac{3}{4}$ 4) $-\frac{4}{3}$

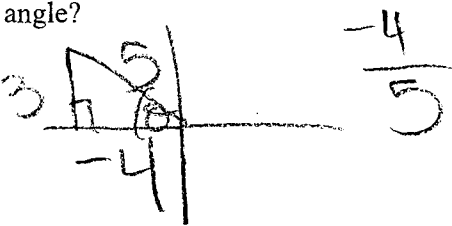


4 If $\sin \theta = -\frac{3}{5}$ and $\cos \theta > 0$, what is the value of $\tan \theta$?

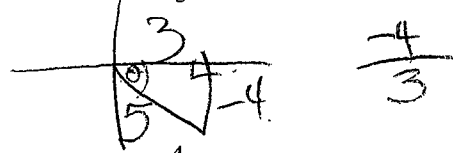
- 1) $\frac{3}{4}$ 2) $-\frac{3}{4}$ 3) $\frac{4}{3}$ 4) $-\frac{4}{3}$



5 If the sine of an angle is $\frac{3}{5}$ and the angle is not in Quadrant I, what is the value of the cosine of the angle?



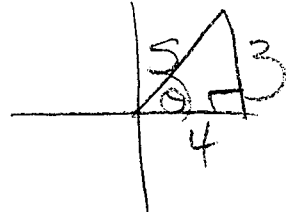
6 If $\sin \theta = -\frac{4}{5}$ and θ is in Quadrant IV, find $\tan \theta$.



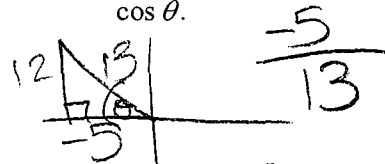
7 If $\cos A = \frac{4}{5}$ and A is in Quadrant I, what is the value of $\sin A \cdot \tan A$?

- 1) $\frac{9}{20}$ 2) $\frac{12}{25}$ 3) $\frac{16}{25}$ 4) $\frac{16}{20}$

$$\frac{3}{5} \cdot \frac{3}{4} = \frac{9}{20}$$

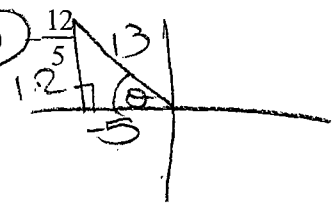


8 If θ terminates in Quadrant II and $\sin \theta = \frac{12}{13}$, find $\cos \theta$.



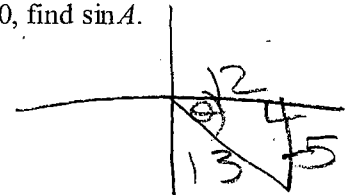
9 If $\cos \theta = -\frac{5}{13}$ and $\sin \theta > 0$, then $\tan \theta$ is

- 1) $\frac{5}{12}$ 2) $-\frac{5}{12}$ 3) $\frac{12}{5}$ 4) $-\frac{12}{5}$

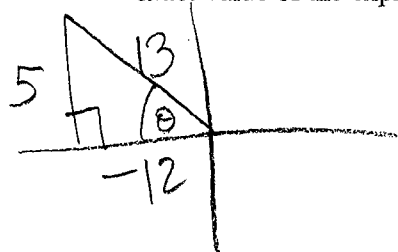


10 If $\tan A = -\frac{5}{12}$ and $\cos A > 0$, find $\sin A$.

$$-\frac{5}{13}$$



11 Given $\tan \theta = -\frac{5}{12}$ and $\frac{\pi}{2} < \theta < \pi$, determine the exact value of the expression $\sin \theta \cot \theta$.



$$\frac{5}{13} \cdot \frac{-12}{5} = -\frac{12}{13}$$

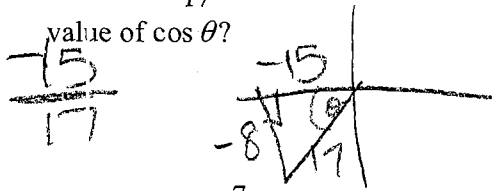
Regents Exam Questions

F.TF.C.8: Determining Trigonometric Functions 1a

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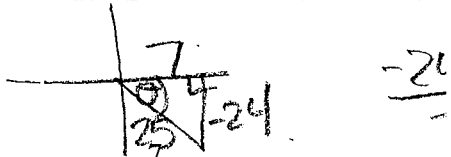
Name: Key

- 12 If $\sin \theta = -\frac{8}{17}$ and $\tan \theta$ is positive, what is the value of $\cos \theta$?



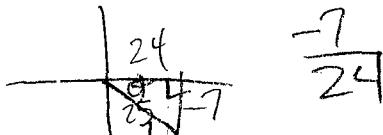
- 13 Given $\cos \theta = \frac{7}{25}$, where θ is an angle in standard position terminating in quadrant IV, and $\sin^2 \theta + \cos^2 \theta = 1$, what is the value of $\tan \theta$?

- 1) $-\frac{24}{25}$ 2) $-\frac{24}{7}$ 3) $\frac{24}{25}$ 4) $\frac{24}{7}$



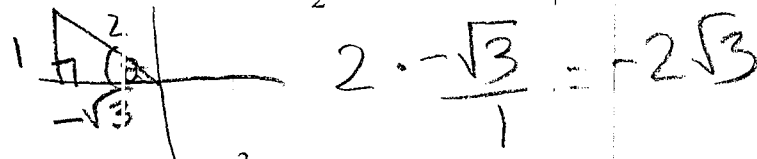
- 14 If $\sin A = -\frac{7}{25}$ and $\angle A$ terminates in quadrant IV, $\tan A$ equals

- 1) $-\frac{7}{25}$ 2) $-\frac{7}{24}$ 3) $-\frac{24}{7}$ 4) $-\frac{24}{25}$



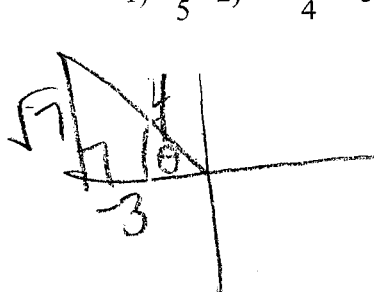
- 15 If $\sin \theta = \frac{1}{2}$ and θ terminates in Quadrant II, what is the value of $\csc \theta \cdot \cot \theta$?

- 1) $2\sqrt{3}$ 2) $\frac{\sqrt{3}}{2}$ 3) -2 4) $\frac{2}{\sqrt{3}}$



- 16 If $\cos \theta = -\frac{3}{4}$ and $\tan \theta$ is negative, the value of $\sin \theta$ is

- 1) $\frac{4}{5}$ 2) $-\frac{\sqrt{7}}{4}$ 3) $\frac{7}{4}$ 4) $-\frac{\sqrt{7}}{4}$



$$\begin{aligned} (-3)^2 + c^2 &= 4^2 \\ 9 + c^2 &= 16 \\ \sqrt{c^2} &= \sqrt{7} \end{aligned}$$

- 17 If $\sin \theta = \frac{\sqrt{7}}{4}$ and $\cos \theta = -\frac{3}{4}$, what is $\tan \theta$?

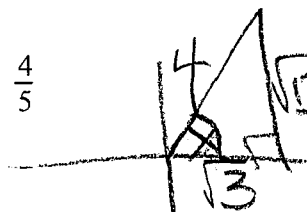
- 1) $\frac{4}{3}$ 2) $-\frac{\sqrt{7}}{4}$ 3) $\frac{\sqrt{7}}{3}$ 4) $-\frac{\sqrt{7}}{3}$



- 18 If x is a positive acute angle and $\cos x = \frac{\sqrt{3}}{4}$, what is the exact value of $\sin x$?

- 1) $\frac{\sqrt{3}}{5}$ 2) $\frac{\sqrt{13}}{4}$ 3) $\frac{3}{5}$ 4) $\frac{4}{5}$

$$\begin{aligned} (13)^2 + x^2 &= 4^2 \\ 3 + x^2 &= 16 \\ \sqrt{x^2} &= \sqrt{13} \end{aligned}$$



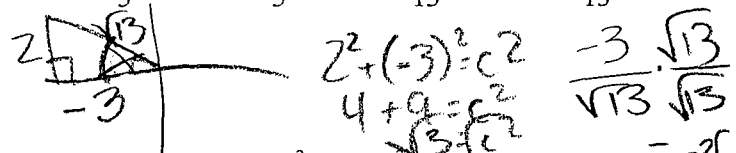
- 19 Given that $\sin^2 \theta + \cos^2 \theta = 1$ and $\sin \theta = -\frac{\sqrt{2}}{5}$, what is a possible value of $\cos \theta$?

- 1) $\frac{5 + \sqrt{2}}{5}$ 2) $\frac{\sqrt{23}}{5}$ 3) $\frac{3\sqrt{3}}{5}$ 4) $\frac{\sqrt{35}}{5}$

$$\begin{aligned} \left(-\frac{\sqrt{2}}{5}\right)^2 + \cos^2 \theta &= 1 \\ \frac{2}{25} + \cos^2 \theta &= 1 \\ \sqrt{\cos^2 \theta} &= \frac{\sqrt{23}}{5} \end{aligned}$$

- 20 If $\tan x = -\frac{2}{3}$ and angle x lies in the second quadrant, what is the value of $\cos x$?

- 1) $\frac{3\sqrt{5}}{5}$ 2) $-\frac{3\sqrt{5}}{5}$ 3) $\frac{3\sqrt{13}}{13}$ 4) $-\frac{3\sqrt{13}}{13}$



- 21 Using the identity $\sin^2 \theta + \cos^2 \theta = 1$, find the value of $\tan \theta$, to the nearest hundredth, if $\cos \theta$ is -0.7 and θ is in Quadrant II.

$$\begin{aligned} \sin^2 \theta + (-0.7)^2 &= 1 \\ \sin^2 \theta + 0.49 &= 1 \\ \sqrt{\sin^2 \theta} &= \sqrt{0.51} \\ \sin \theta &= \sqrt{0.51} \end{aligned}$$

$$\tan \theta = \frac{\sqrt{0.51}}{-0.7} \approx -1.02$$

Trigonometric Identities

(you must memorize)

<p>Reciprocal Identities</p> $\cot \theta = \frac{1}{\tan \theta}$ $\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$	<p>Quotient Identities</p> $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$
<p>Pythagorean Identities</p> $\sin^2 \theta + \cos^2 \theta = 1$ $\tan^2 \theta + 1 = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$	

Name: Key

$$\sin^2 \theta + \cos^2 \theta = 1$$

F.TF.C.8: Simplifying Trigonometric Expressions

1 If $\sin^2(32^\circ) + \cos^2(M) = 1$, then M equals

- 1) 32°
- 2) 58°
- 3) 68°
- 4) 72°

2 Which expression always equals 1?

- 1) $\cos^2 x - \sin^2 x$
- 2) $\cos^2 x + \sin^2 x$
- 3) $\cos x - \sin x$
- 4) $\cos x + \sin x$

3 The expression $\cos^2 4\theta + \sin^2 4\theta$ is equivalent to

- 1) 1
- 2) 2
- 3) $\cos \theta$
- 4) $\cos 8\theta$

4 The expression $\frac{\sin^2 x + \cos^2 x - b^2}{1 - b^2}$ is equivalent to

- 1) 1
- 2) b^2
- 3) $(1+b)(1-b)$
- 4) $\sin x \cos x - b$

5 If θ is a positive acute angle and $\sin \theta = a$, which expression represents $\cos \theta$ in terms of a ?

- 1) \sqrt{a}
 - 2) $\sqrt{1-a^2}$
 - 3) $\frac{1}{\sqrt{a}}$
 - 4) $\frac{1}{\sqrt{1-a^2}}$
- $\sin^2 \theta + \cos^2 \theta = 1$
 $\sqrt{\cos^2 \theta} = \sqrt{1-a^2}$
 $\cos \theta = \sqrt{1-a^2}$

6 If $\sin A = k$, then the value of the expression $(\sin A)(\cos A)(\tan A)$ is equivalent to

- 1) 1
- 2) $\frac{1}{k}$
- 3) k
- 4) k^2

$k \cdot \cos A \cdot \frac{\sin A}{\cos A}$
 $k \cdot \sin A$
 $k \cdot k$
 k^2

7 The expression $\frac{\cos^2 x}{1 - \sin^2 x}$ is equivalent to

- 1) 1
- 2) -1
- 3) $\cos x$
- 4) $\sin x$

8 The expression $\frac{1 - \cos^2 x}{\sin^2 x}$ is equivalent to

- 1) 1
- 2) -1
- 3) $\sin x$
- 4) $\cos x$

9 The expression $\frac{\sin^2 A}{\tan A}$ is equivalent to

- 1) $\frac{\sin A}{\cos A}$
- 2) $\sin A \cos A$
- 3) $\frac{1}{\sin A \cos A}$
- 4) $\frac{\cos A}{\sin A}$

$\frac{\sin^2 A}{\sin A \cos A} = \frac{\sin A}{\cos A} = \sin A \cos A$

10 The expression $\frac{\sin x \cdot \cos x}{\tan x}$ is equivalent to

- 1) 1
- 2) $\sin^2 x$
- 3) $\cos x$
- 4) $\cos^2 x$

$\frac{\sin x \cdot \cos x}{\frac{\sin x}{\cos x}} = \sin x \cdot \cos x \cdot \frac{\cos x}{\sin x} = \cos^2 x$

11 Express in simplest terms: $\frac{2 - 2\sin^2 x}{\cos x}$

$\frac{2(1 - \sin^2 x)}{\cos x} = \frac{2 \cos^2 x}{\cos x} = 2 \cos x$

12 For all values of θ for which the expressions are defined, prove that the following is an identity:

$\cos \theta (\cos \theta + 1) + \sin^2 \theta = \frac{\sin \theta + \tan \theta}{\tan \theta}$

$\cos^2 \theta + \cos \theta + \sin^2 \theta$
 $1 + \cos \theta = \frac{\sin \theta + \frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}}$
 $\frac{\sin \theta + \frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} = \frac{\sin \theta \cos \theta + \sin \theta}{\sin \theta} = \cos \theta + 1$